

The last chapter (Chapter 6) is mainly devoted to applications to nonlinear integral equations via linearization by Newton's method.

There are two appendices. The first one by the author summarizes basic properties of compact operators and provides a convenient reference for reading this book. The second appendix, by Joel Davis, discusses practical implementation of some of the methods in this book and provides numerical examples (Table 1 to Table 6). There is a bibliography at the end of the book which cites 99 references.

As a whole, this book gives, in a relatively few pages (136 pages), a first-hand account of essential parts of collectively compact operator theory which have been developed in the past decade, mainly by the author and his colleagues.

This book may be used in an advanced graduate course or in a research seminar. In the former, the instructor may feel the need to supplement his lecture with appropriate exercises since no exercises are provided in this book.

YASUHIKO IKEBE

Center for Numerical Analysis  
The University of Texas  
Austin, Texas 78712

34[7].—P. F. BYRD & M. D. FRIEDMAN, *Handbook of Elliptic Integrals for Engineers and Scientists*, Springer-Verlag, New York, 1971, xvi + 358 pp., 24 cm. Price \$18.50.

The first edition of this volume appeared in 1954 and, to my surprise, I find that it was never reviewed in these annals. This valuable and useful tome I am sure is quite well known. Nonetheless, some descriptive remarks are in order. Its title is a good summary of its contents. It contains over 3000 important formulas to facilitate the reduction and evaluation of elliptic integrals. Also included are short tables of the elliptic integrals of the first and second kind, Jacobi's  $q$ -function, Heuman's  $\Lambda_0$ -function and Jacobi's ( $K$ -multiplied) zeta-function.

The second edition is unfortunately not much different from the first. The authors recognize that since 1954 numerous computational methods for efficient calculation of elliptic integrals and Jacobian elliptic functions have been published. Further, an abundance of new tabular material has appeared. To improve the first edition by augmenting its contents to include this material would, in the view of the authors, necessitate another volume and must be deferred. Thus, the first edition is reproduced essentially without change, except for a supplementary bibliography and corrections. Unfortunately, the bibliography is incomplete, especially with regard to tabular material, and not all errors noted in the first edition have been corrected.

Errata to the first edition have been recorded in these annals [1], [2], [3], [4]. We have examined these data against the new edition and find some errors still persist. For example, consider the evaluation of

$$\Pi(\phi, \alpha^2, 1) = \int_0^\varphi \frac{\alpha \theta}{(1 - \alpha^2 \sin^2 \theta) \cos \theta}.$$

In the first edition, on p. 10, the third line of formula 111.04 reads

$$(1) \Pi(\phi, \alpha^2, 1) = \frac{1}{1 - \alpha^2} \left[ \ln(\tan \phi + \sec \phi) - \alpha \ln \left( \frac{1 + \alpha \sin \phi}{1 - \alpha \sin \phi} \right)^{1/2} \right], \quad \alpha^2 \neq 1.$$

In [2, p. 533], H. E. Fettis remarked that for the validity of (1), one should have  $\alpha^2 > 0$  and  $\alpha \sin \phi < 1$ . He suggested that (1) be replaced by two equations. Thus,

$$(2) \Pi(\phi, \alpha^2, 1) = \frac{1}{1 - \alpha^2} \left[ \ln(\tan \phi + \sec \phi) - \alpha \ln \left( \frac{1 + \alpha \sin \phi}{1 - \alpha \sin \phi} \right)^{1/2} \right], \quad \alpha^2 > 0,$$

$$(3) \Pi(\phi, \alpha^2, 1) = \frac{1}{1 - \alpha^2} [\ln(\tan \phi + \sec \phi) + |\alpha| \arctan (|\alpha| \sin \phi)], \quad \alpha^2 < 0.$$

In the second edition, (3) is given, but, in place of (2), we find (1) with the added restriction,  $\alpha^2 \neq 1$ . Clearly, if  $\phi$  is not an odd multiple of  $\pi/2$ , (1) can be evaluated for  $\alpha^2 = 1$  with the aid of L'Hospital's theorem. Actually, for the validity of (2) and (3), we should add the restrictions  $\phi$  real and  $\alpha \sin \phi \neq 1$  with the further understanding that the integral is a Cauchy principal value integral if  $\alpha \sin \phi > 1$ . The conditions are sufficient, since we can easily continue the integral into the complex plane for complex values of  $\alpha$  and  $\phi$ . Note that for  $\phi$  real, we can use the restriction  $0 < \phi < \pi/2$  in view of periodicity and the fact that the integral has the same value for  $\phi = \pi/2 \pm \omega, 0 < \omega \leq \pi/2$ .

In the tables for  $KZ(\beta, k)$ , the following corrections should be made in both editions:

Page	$\sin^{-1} k$	$\beta$	for	read
339	15°	44°	.027204	.027203
340	40°	73°	.124059	.124061

These as well as others were reported in [3, p. 639], and all have been corrected in the second edition except those just noted.

An error noted in [4] has not been corrected in the second edition. On p. 289, Formula 800.07 (same in both editions), the third term in the third line should be  $-\pi K'/2$  instead of  $+\pi K'/2$ . In the first edition, the upper limit of the first integral in this formula was given incorrectly as  $K$ . It should read 1. This error was noted in [4] and has been corrected in the second edition.

Some further errors have been noticed by H. E. Fettis and appear in the errata section of this issue.

Y. L. L.

1. *MTAC*, v. 13, 1959, p. 141.
2. *Math. Comp.*, v. 18, 1964, pp. 533, 687.
3. *Math. Comp.*, v. 20, 1966, pp. 344, 639.
4. *Math. Comp.*, v. 23, 1969, p. 468.